Solution exercice 40

$$V_0 + V_1 + V_2 + \cdots + V_n = (n+1)\cdot\left(\frac{V_0 + V_n}{2}\right)$$

$$= (n+1)\cdot\left(\frac{3 + V_0 + \pi . n}{2}\right)$$

$$= (n+1)\cdot\left(\frac{3 + 3 + 2n}{2}\right)$$

$$= (n+1)\left(\frac{6+2n}{2}\right)$$

$$= (n+1)\cdot(3+n)$$

Solution exercice 41

$$V_n = U_n - \frac{1}{4} \Rightarrow V_{n+1} = U_{n+1} - \frac{1}{4} = \frac{1}{3}U_n + \frac{1}{6} - \frac{1}{4}$$

$$= \frac{1}{3}U_n - \frac{1}{12} = \frac{1}{3} V_n$$

(V_n) géo de raison $q = \frac{1}{3}$ et $V_0 = U_0 - \frac{1}{4} = \frac{3}{4}$

$$S_n = V_0 + V_1 + \cdots + V_n = V_0 \cdot \frac{1-q^{n+1}}{1-q}$$

$$= \left(\frac{V_0}{1-q}\right)\cdot\left(1 - q^{n+1}\right) = \left(\frac{3/4}{1-\frac{1}{3}}\right)\left(1 - \left(\frac{1}{3}\right)^{n+1}\right)$$

$$= \frac{9}{8}\left(1 - \left(\frac{1}{3}\right)^{n+1}\right)$$

On a : $V_n = U_n - \frac{1}{4} \Rightarrow U_n = V_n + \frac{1}{4}$

$$T_n = U_0 + U_1 + U_2 + \cdots + U_n$$

$$= \left(V_0 + \frac{1}{4}\right) + \left(V_1 + \frac{1}{4}\right) + \left(V_2 + \frac{1}{4}\right) + \cdots + \left(V_n + \frac{1}{4}\right)$$

$$= \left(V_0 + V_1 + \cdots + V_n\right) + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \cdots + \frac{1}{4}$$

$$= \frac{9}{8}\left(1 - \left(\frac{1}{3}\right)^{n+1}\right) + \frac{1}{4}\cdot(n+1)$$